Algebraic Bayesian analysis of contingency tables with possibly zero-probability cells

Guido Consonni and Giovanni Pistone

University of Pavia, Italy and Politecnico di Torino, Italy

Abstract: In this paper we consider a Bayesian analysis of contingency tables allowing for the possibility that cells may have probability zero. In this sense we depart from standard log-linear modeling that implicitly assumes a positivity constraint. Our approach leads us to consider mixture models for contingency tables, where the components of the mixture, which we call model-instances, have distinct support. We rely on ideas from polynomial algebra in order to identify the various model instances. We also provide a method to assign prior probabilities to each instance of the model, as well as describing methods for constructing priors on the parameter space of each instance. We illustrate our methodology through a $5 \times 2$ table involving two structural zeros, as well as a zero count. The results we obtain show that our analysis may lead to conclusions that are substantively different from those that would obtain in a standard framework, wherein the possibility of zero-probability cells is not explicitly accounted for.

Key words and phrases: Algebraic statistics; Bayes factor; Compatible priors; Exponential family; Log-linear model; Model-instance; Positivity constraint; Structural zero; Toric model.